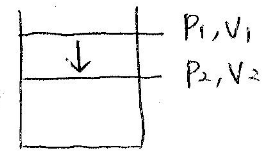
2007 後期第2回

[1] 熱力学第1法則

$$\Delta U = \Delta Q + \Delta W$$

$dU = \delta Q + \delta W$

(a) 等温圧縮 $\Delta T = 0$



内部エネルギー $U = nCT$

仕事と熱は全て交換する

$$dU = nC\,dT = \delta W + \delta Q = 0$$

$$\Delta Q = -\Delta W = P\Delta V$$

気体が吸収する熱量 $\Delta Q = \int_{V_1}^{V_2} dQ$

$$= \int_{V_1}^{V_2} P(V, \underset{\text{const}}{T})\, dV$$

$$= \int_{V_1}^{V_2} nRT/V \, dV = nRT \ln \frac{V_2}{V_1}$$

(b) * $V = \text{const}$

$$\Delta Q + \underset{\substack{\| \\ 0}}{\Delta W} = \Delta U = nC\Delta T$$

$$\left(\frac{\Delta Q}{\Delta T}\right)_V = C_V = \underset{\approx 1}{n}C = C$$

$n \approx 1$ (モル比熱なので) ⇒ CはC_Vと一致する

* $P = \text{const}$

$$\Delta Q + \Delta W = \Delta U = nC\Delta T, \quad \Delta Q = P\Delta V + nC_V\Delta T$$

$n = 1$

$$C_p = \left(\frac{\Delta Q}{\Delta T}\right)_p = P\underbrace{\left(\frac{\Delta V}{\Delta T}\right)_p}_{\Delta V/\Delta T = R/P} + C_V = R + C_V \quad \therefore C_p - C_V = R$$

(c) 断熱. $\delta Q = 0$.

$$dW = dU = -p\,dV.$$

$$dU = C_V\,dT.$$

$$\rightarrow \quad C_V\,dT + p\,dV = 0. \quad \cdots (1)$$

1モル ideal gas. $PV = RT$.

$$p\,dV + V\,dp = R\,dT. \qquad \text{(b)より } R = C_p - C_V$$

$$\therefore\ dT = \frac{p\,dV + V\,dp}{C_p - C_V}.$$

(1)に代入して,

$$\frac{C_V}{C_p - C_V}(p\,dV + V\,dp) + p\,dV = 0.$$

$$\cancel{C_V p\,dV} + C_V V\,dp + C_p p\,dV - \cancel{C_V p\,dV} = 0.$$

$$pC_p\,dV + VC_V\,dp = 0.$$

$$\frac{dp}{p} + \gamma \frac{dV}{V} = 0. \qquad \left(\frac{C_p}{C_V} = \gamma\right).$$

両辺積分.

$$\ln p + \gamma \ln V = \text{const}.$$

$$\ln pV^{\gamma} = \text{const}.$$

$$\therefore\ pV^{\gamma} = \text{const}. \qquad (\gamma = C_p/C_V).$$

$PV = RT$を用いると,

$$TV^{\gamma-1} = \text{const}.$$

$$T^{\gamma} p^{1-\gamma} = \text{const}. \quad \text{も成り立つ。}$$

(d) 等積加熱. $\Delta V = 0$.

$$\Delta Q + \underbrace{\Delta W}_{=0} = \Delta U = nC_v \Delta T,$$

$$Q = \int_{T_1}^{T_2} dQ = \int_{T_1}^{T_2} nC_v \, dT = nC_v (T_2 - T_1)$$

e

(~~d~~) $(P_1, V_1) \to (P_2, V_2)$.

(i) 断熱圧縮 $\Delta Q = 0$.

$\Delta U = \Delta W$.

* 気体にされた仕事. $W_1 = \int_{V_1}^{V_2} dW = \int_{V_1}^{V_2} dU$

$= \int_{T_1}^{T_2} nC_v dT = nC_v(T_2 - T_1)$

* 吸収した熱量 $Q_1 = 0$.

(ii) 等温圧縮. $\Delta T = 0$.

$\Delta Q + \Delta W = \Delta U = nC_v \Delta T = 0$.

$\Delta Q = -\Delta W$.

(a) より $\Delta Q = nRT \ln \frac{V_2}{V_1}$

$\Delta W = -nRT \ln \frac{V_2}{V_1}$.

等積冷却. $\Delta V = 0$, $\Delta W = 0$.

(d) より $\Delta Q = nC_v(T_2 - T_1)$.

$\Delta W = 0$.

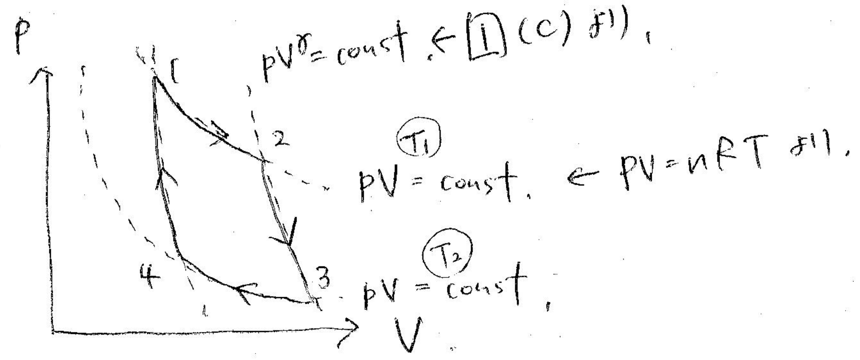
~~等温加熱. $\Delta T = 0$.~~

$\therefore Q_2 = nRT \ln \frac{V_2}{V_1} + nC_v(T_2 - T_1)$

$W_2 = -nRT \ln \frac{V_2}{V_1}$.

(i)(ii) より、 $Q_1 + W_1 = Q_2 + W_2$ $(= nC_v(T_2 - T_1))$.

[2] (a)



1→2 等温膨張
2→3 断熱膨張
3→4 等温圧縮
4→1 断熱圧縮

(b) 等温変化 $\Delta T = 0$.

$$\Delta Q + \Delta W = 0,$$

$$\Delta Q = +p\,dV = nRT_1 \ln \frac{V_2}{V_1} = Q_1$$

$$nRT_2 \ln \frac{V_4}{V_3} = Q_2.$$

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = nRT_1 \ln\left(\frac{V_2}{V_1}\right) \Big/ T_1 + nRT_2 \ln\left(\frac{V_4}{V_3}\right) \Big/ T_2$$

$$= nR\left(\ln \frac{V_2}{V_1} + \ln \frac{V_4}{V_3}\right)$$

$$= nR \ln \frac{V_2 V_4}{V_1 V_3} \quad \cdots (i)$$

ここで、

$$\begin{cases} p_1 V_1 = p_2 V_2 & \cdots ① \\ p_3 V_3 = p_4 V_4 & \cdots ② \end{cases}$$

$$\begin{cases} p_2 V_2^{\gamma} = p_3 V_3^{\gamma} & \cdots ③ \\ p_4 V_4^{\gamma} = p_1 V_1^{\gamma} & \cdots ④ \end{cases}$$

の関係になっているので、

①②より $V_2/V_1 = p_1/p_2$, $V_3/V_4 = p_4/p_3$.

③④と合わせて、 $$\frac{V_2V_4}{V_1V_3} = \frac{p_1}{p_2}\frac{p_3}{p_4} = \frac{p_1 p_3}{p_3 V_3^{\gamma} V_2^{-\gamma}\, p_1 V_1^{\gamma} V_4^{-\gamma}} = \left(\frac{V_2V_4}{V_1V_3}\right)^{\gamma}$$

$$\rightarrow \left(\frac{V_2V_4}{V_1V_3}\right)^{\gamma-1} = 1. \qquad \therefore V_2/V_1 = V_3/V_4$$

$\therefore$ (i)式 $= 0$.

$$\therefore Q_1/T_1 + Q_2/T_2 = 0$$

(c) 有効熱 $Q = Q_1 - Q_2$.

$\xi = (Q_1 - Q_2)/Q_1$

変換率 $\xi = W/(Q_1 - Q_2)$　　$\therefore \overset{効率}{\eta} = W/Q_1$.

~~(d)~~ 外から吸収する熱: $Q_{1\to2}$.

$$Q_{1\to2} = nRT_1 \ln\left(\frac{V_2}{V_1}\right)$$

外にする仕事 :

$$W_{1\to2} = nRT_1 \ln\left(\frac{V_2}{V_1}\right)$$

$$W_{2\to3} = nC_V(T_2 - T_1)$$

$$W_{3\to4} = nRT_2 \ln\left(\frac{V_4}{V_3}\right) = -nRT_2 \ln\left(\frac{V_2}{V_1}\right).$$

$$W_{4\to1} = nC_V(T_1 - T_2)$$

$$\rightarrow W = nR(T_1 - T_2)\ln(V_2/V_1).$$

$$\eta = W/Q_{1\to2} = (T_1 - T_2)/T_1 = 1 - \frac{T_2}{T_1}$$

・100%の熱効率はあり得ない。
・Carnot Cycleの効率は熱源の温度だけで決まる

(d) 最大効率は

$\eta = 1 - \frac{T_2}{T_1}$ で与えられるので,

$T_2 = 27°C = 300K$,
$T_1 = 327°C = 600K$.　　$\eta = 1 - \frac{300}{600} = 0.5$,

効率 50%

3 (a) 等温：$T = \text{const}$.

$p = \frac{\sigma}{3} T^4$

$U = \sigma V T^4$.

$$\Delta Q = P\Delta V + \Delta U.$$
$$= \frac{\sigma T^4}{3}\Delta V + \sigma T^4 \cdot \Delta V$$
$$= \frac{4}{3}\sigma T^4 \cdot \Delta V$$

~~(b) 断熱：$\Delta Q = 0$.~~

~~$\rightarrow P\Delta V + \Delta U = 0$~~

吸熱量 $Q = \int_{V_1}^{V_2} \frac{4}{3}\sigma T^4 dV = \frac{4}{3}\sigma T^4 (V_2 - V_1)$

外にした仕事 $W = \int_{V_1}^{V_2} \underbrace{p\,dV}_{\Delta W} = \int_{V_1}^{V_2} \frac{\sigma T^4}{3} dV = \frac{\sigma}{3} T^4 (V_2 - V_1)$

(b) 断熱：$\Delta Q = 0$.

$3PV = U$.

$\Delta U = 3P\Delta V + 3\Delta P V$.

$$P\Delta V + \Delta U = 0.$$
$$P\Delta V + 3P\Delta V + 3\Delta P V = 0.$$
$$4P\Delta V = -3\Delta P V$$
$$\int \frac{\Delta P}{P} = -\frac{4}{3}\int \frac{\Delta V}{V}$$
$$\log P = \log V^{-4/3} + C.$$
$$\therefore PV^{4/3} = \text{const}. = A \text{ とする.}$$

$p = \text{const}$?

ΔP は動くのか？ U が動くと

~~$P\Delta V + \Delta U = 0.$~~

~~$\frac{\sigma}{3}T^4 \Delta V + \sigma \Delta V T^4 = 0.$~~

$\neq 0$

$T^4 V^{4/3}$.

$TV^{1/3} = \text{const}$.

$\boxed{VT^3 = \text{const}}$

外にする仕事 $W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{A}{V^{4/3}} dV$

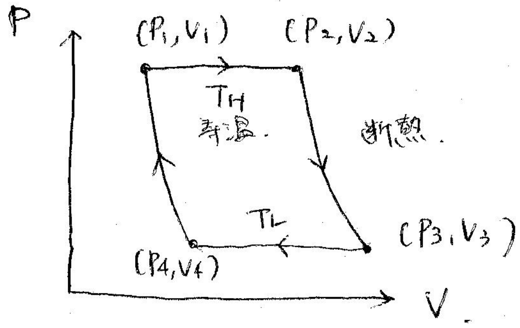
$$= A \cdot -3 \left(V_2^{-1/3} - V_1^{-1/3}\right)$$
$$= 3A\left(V_1^{-1/3} - V_2^{-1/3}\right)$$

$A = PV^{4/3}$.

$p = \frac{\sigma}{3}T^4$.

$$= 3 \cdot PV^{4/3}\left(V_1^{-1/3} - V_2^{-1/3}\right)$$
$$= 3(P_1 V_1 - P V_2) \quad = \sigma\left(V_1 T_1^4 - V_2 T_2^4\right)$$

(c) Carnot cycle.



効率を求める： 効率 $\eta = \frac{W}{Q} = \frac{\text{した仕事}}{\text{吸収した熱量}}$

吸収した熱量： $Q = \frac{4}{3}\sigma T_H^4 (V_2 - V_1)$ ← (a)より：等温膨張時．断熱のときは熱の出入りなし．

仕事：
$$W = \underset{(1\to 2)}{\frac{\sigma}{3} T_H^4 (V_2 - V_1)} + \underset{(2\to 3)}{\sigma (V_2 T_H^4 - V_3 T_L^4)}$$
$$+ \underset{(3\to 4)}{\frac{\sigma}{3} T_L^4 (V_4 - V_3)} + \underset{(4\to 1)}{\sigma (V_4 T_L^4 - V_1 T_H^4)}$$
$$= \frac{\sigma}{3} T_H^4 V_2 - \frac{\sigma}{3} T_H^4 V_1 + \sigma T_H^4 V_2 \boxed{- \sigma T_L^4 V_3}$$
$$+ \frac{\sigma}{3} T_L^4 V_4 \boxed{- \frac{\sigma}{3} T_L^4 V_3} + \sigma T_L^4 V_4 - \sigma T_H^4 V_1$$

2)(b)より．

$$\boxed{V T^3 = \text{const.}}$$

$$T_L^3 V_3 = T_H^3 V_2$$
$$T_L^3 V_4 = T_H^3 V_1$$

$$= \frac{4}{3}\sigma T_H^4 V_2 - \frac{4}{3}\sigma T_L^4 V_3 - \frac{4}{3}\sigma T_H^4 V_1 + \frac{4}{3}\sigma T_L^4 V_4$$
$$= \frac{4}{3}\sigma \{ T_H^4 (V_2 - V_1) + T_L^4 (V_4 - V_3) \}$$
$$= \frac{4}{3}\sigma (V_2 - V_1) \cdot T_H^3 (T_H - T_L)$$

$$\therefore \frac{W}{Q} = \frac{T_H^4 - T_H^3 T_L}{T_H^4} = 1 - \frac{T_L}{T_H}$$

$\cancel{U_3 = \sigma V_3 T_L^4}$
$\cancel{U_2 = \sigma V_2 T_H^4}$
$\cancel{U_3 - U_2 = \sigma (V_3 T_L^4 - V_2 T_H^4)}$